# A New Method of Estimating Volume During Powder Compaction and the Work of Compaction on a Rotary Tablet Press from Measurements of Applied Vertical Force

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Abstract-The volume per unit mass of a powder bed, V', during compaction on a rotary tablet press has **Abstract**—The volume per unit mass of a powder bed, V', during compaction on a rotary tablet press has been expressed as a function of pressure, P using a modification of Kawakita's equation:  $V = (V_o' - V')P'/(P+P') + V'$ , where specified tableting conditions. The volume, V, is determined from the machine deformation constant which is the relationship between applied vertical force and the deformation of the tablet press and the punches. An iterative method is described which allows the determination of  $V_0$ , V' and P' from the slope and intercept of V vs  $1/(P+P')$  where all values are evaluated at peak pressure. By substituting these values into the equation, the volume of a given powder bed during compaction up to peak pressure can be accurately predicted from the pressure vs time curve. This method of estimating volume and hence punch displacement, is much simpler than an earlier analytical method which was derived from direct measurements of punch displacement under running conditions. Since volume is an explicit function of pressure, the work of compaction is also a function of pressure. Estimates of the work of compaction are in good agreement with values calculated using our previous method. Values of  $V_0$ , V' and P' are reported for **35** pharmaceutical materials and could be incorporated into a database library of drugs and tableting excipients. This database could then be used for the quality control of incoming raw materials (batch to batch assessment) and for the comparison of materials from alternative sources. The experimental methodology and method of calculation should, in principle, be applicable to any rotary tablet press and together with other tableting parameters (such as compression time, peak offset time, decompression time, elastic recovery and work of compaction) would provide a simple, inexpensive method for the in process validation of tablet compression.

Numerous equations have been proposed which give a relationship between the volume of a powder bed during compaction in a punch and die assembly and punch pressure (MacLeod 1983). The equation most widely used in the pharmaceutical literature is due to Heckel (1961) which can be expressed as:

$$
\ln\left(\frac{V}{V-V_{\infty}}\right) = c_1 P + \ln c_2 \tag{1}
$$

where  $V =$  the powder volume under an applied pressure, P,  $V_{\infty}$  = the volume of the compact when its porosity is zero, and  $c_1$  and  $c_2$  are constants. Where a straight line relationship exists between  $ln(V/(V - V_{\infty}))$  and P, the reciprocal of the slope,  $1/c<sub>1</sub>$ , is said to be numerically equal to the mean yield stress,  $P_v$ , (Hersey & Rees 1971; Roberts & Rowe 1987). Unfortunately, many reported estimates of  $P<sub>v</sub>$  are questionable. The slope is rarely linear and, all too frequently, the volume between the punch faces has been incorrectly estimated because either the linear variable differential transformers (LVDTs) used to measure the distance between the punch faces have been incorrectly mounted, or the deformation in the punches and the press whilst under load has been ignored.

Kawakita & Ludde (1970/71) related the volume of a powder bed to the applied pressure. One form of Kawakita's equation was written:

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$$
\left(P + \frac{1}{b}\right)(V - V_{\infty}) = \frac{V_o - V_{\infty}}{b}
$$
 (2)

where  $V_0$  = the initial apparent powder volume and b is a constant. Equation 2, which is similar in form to that of van der Waal's equation of state for gases, can be expressed:<br>  $(P + P'') (V - V_x) = (V_0 - V_x) P''$  (3)

$$
P + P''
$$
)  $(V - V_{\infty}) = (V_o - V_{\infty}) P''$  (3)

where  $1/b = P''$  and all volumes are normalized for powder mass. Rearranging to express volume as a function of pressure gives:

$$
P = (V_o - V_{\infty}) \cdot P''/(V - V_{\infty}) - P''
$$
 (4)

which implies a linear relationship between P and  $1/(V - V_\infty)$  with a slope of  $(V_o - V_\infty)P''$  and an intercept of  $- P''$ . The values of  $V_0$  and  $P''$  are not material constants but are constants for a given powder bed. Both  $V_0$  and P'' vary with such factors as particle size, shape and roughness which affect the initial packing.

Heckel's equation and Kawakita's equation, both model the P-V relationship from  $P = 0$  up to  $P = \infty$ , where the corresponding volumes are  $V_0$  and  $V_\infty$ , respectively. The volume,  $V_{\infty}$ , is the minimum theoretical volume of the compact per unit mass (i.e.  $V = V_{\infty}$  when the porosity of the compact  $= 0$ ).

Using a Manesty Betapress, we previously evaluated the compaction profiles of a wide range of pharmaceutical materials using a novel method of estimating punch displacement from measurements of applied vertical force and turret position only (Oates & Mitchell 1989, 1990; Dwivedi et a1 1991, 1992). The compression data has now been reanalysed using Kawakita's equation and modifications thereof which take elastic deformation in the press, punches and die into account. The analysis of punch displacement is much simpler than our previous method and requires only an accurate determination of the dependence of press and punch deformation on applied vertical force under static conditions to give a machine deformation constant,  $K_m^{-1}$ .

## **Materials and Methods**

The instrumentation of the Betapress and the methods of data collection were as described previously. Flat-faced 1/2" (1.270 cm) IPT tooling operated at a turret revolution time of 1 **s** was used throughout. For a given material,  $V<sub>x</sub>$  in equation **3** is a constant and was calculated from the true density of the solid:

$$
\mathbf{V}_{\infty} = 1/\rho \tag{5}
$$

The values of  $\rho$ , as determined by helium-displacement pycnometry, are given in Table 1 or were taken from Dwivedi et a1 (1992). Oates & Mitchell (1989) showed that the Betapress undergoes reversible elastic deformation when a vertical force is applied and that the extent of deformation is proportional to the applied force. At peak force,  $F_{\text{max}}$ , the tablet thickness per unit mass,  $H_p$ , is given by:

$$
H_p = (H_o + K_m^{-1} \cdot F_{max})/mass
$$
 (6)

where  $H_0$  = the minimum distance between the upper and lower punch faces in an empty die and,  $K_m^{-1}$  = the machine deformation constant =  $2.3 \times 10^{-6}$  cm N<sup>-1</sup>. The tablet thickness setting was fixed to give  $H_0 = 0.314$  cm (Dwivedi et a1 1992). If it is assumed that die expansion is negligible when the radial force is applied, then the volume of the compact per unit mass at peak pressure,  $V_p$ , is given by:

$$
V_p = H_p \cdot A_o \tag{7}
$$

where  $A_0$  = the cross-sectional area of the die when the radial pressure, P,, equals zero.

Table I. True densities of various pharmaceutical materials'.

	Density	
Material	$(g \text{ cm}^{-3})$	Manufacturer
Calcium phosphates		
$A-Tab$	2.774	Rhone-Poulenc
<b>Anhydrous Emcompress</b>	2.780	Edward Mendell
CalStar	2.316	<b>FMC</b> Corporation
Di-Tab	2.330	Rhone-Poulenc
Tri-Tab	2.883	Rhone-Poulenc
Lactose		
Anhydrous	1.564	Sheffield
<b>DCL 21</b>	1.561.	De Melkindustrie Veghel
Fast-flo	1.553	Foremost
Microcrystalline cellulose		
Avicel Large	1.555	
Avicel PH101	1.556	<b>FMC</b> Corporation <b>FMC</b> Corporation
Avicel PH105	1.556	<b>FMC</b> Corporation
<b>Sugars</b>		
Emdex	1.504	Edward Mendell
Mannitol MG	1.482	Roquette
Neosorb	1.487	Roquette
Xylitol	1.533	Roquette

\* See Dwivedi et al (1992) for true densities of other materials.

### **Results and Discussion**

Equation 4 was evaluated at peak pressure where  $P = P_{max}$ and  $V = V_p$ . Fig. 1 shows typical plots of P vs  $1/(V - V_\infty)$ for various solids where P<sub>max</sub> ranged from about 25 to 250 MPa. For some materials (e.g. anhydrous Emcompress, paracetamol and sucrose) the relationship is linear but for others (e.g. Avicel PH102, Emdex and STA-Rx-1500) there is distinct curvature. Table 2 gives the correlation coefficients and values of P' determined from the intercept of the line of best fit for a wide range of materials. As expected the error in P' increases with the increase in nonlinearity.

Equation **4** contains certain assumptions which contribute to the anomalous values shown in Table *2.* Unlike gases, the pressure is not applied hydrostatically. The applied axial pressure produces an unequal pressure in the radial direction. The die undergoes radial expansion thereby introducing errors in the estimation of  $V_p$ . This was particularly noticeable with aspirin and ibuprofen where, if die expansion is assumed to be negligible, the calculated porosity goes to less than zero above a certain pressure.

In an attempt to obtain a better fit, some of the terms in equation 3 were modified to give an equation which takes radial expansion of the die into account. At peak pressure the term  $V - V_{\infty}$  was written:

$$
V - V_{\infty} = V_{p} - V_{\infty} = (H_{p} - V_{\infty}/A(P_{r})) A_{o}
$$
  
=  $(H_{p} - H_{\infty}(P_{r})) A_{o}$  (8)

where  $A =$  the true cross-sectional area of the die as a function of the radial pressure,  $P_r$ , and  $H_\infty$  = the minimum theoretical compact thickness per unit mass in the limit where P goes to infinity and the porosity of the compact  $= 0$ .



FIG. 1. Plots of P<sub>max</sub> vs  $1/(V-V_{\infty})$  according to equation 4.  $\Box$  Anhydrous Emcompress, 0 sucrose,  $\triangle$  paracetamol powder. b. 0 Emdex, **v** Avicel PH102, *0* STA-Rx-1500.



	<b>p</b> "	$V_{o}$	$\mathbf{V}_{\infty}$		
Material Aspirin	(MPa) a	$(cm^3 \frac{9}{2}^{-1})$ a	$(cm^3 \tilde{g}^{-1})$ 0.7172	$r^2$	Material Aspirin
Calcium phosphates A-Tab CalStar Di-Tab Emcompress anhydrous Tri-Tab	175(9) 98(11) 104(8) 109(10) 173(9) 258 (12)	0.61(0.01) 0.52(0.02) 0.52(0.02) 0.52(0.02) 0.61(0.02) 0.57(0.02)	0.3605 0-4318 0.4291 0.4250 0.3598 0.3468	0.9787 0.9737 0.9812 0.9746 0.9789 0.9693	Calcium A-Tal CalSta Di-Ta Emco anh $\operatorname{\sf Tri-Tz}$
Ibuprofen Crystalline <b>DCI-63</b>	a a	a a	0.8932 0.8112		Ibuprofe Crysta $DCI-6$
Lactose Anhydrous <b>DCL 21</b> Fast-flo Monohydrate Spray-dried	27(7) 26(7) 8(7) 46(7) 26(5)	0.97(0.02) 0.92(0.02) 1.91(0.02) 0.69(0.01) 0.85(0.01)	0.6394 0.6406 0.6523 0.6502 0.6502	0.9864 0.9863 0.9853 0.9878 0.9944	Lactose Anhyo $DCL$ : Fast-f Mono Spray
Microcrystalline cellulose Avicel Large Avicel PH101 Avicel PH102 Avicel PH105 Emcocel	$-20(10)$ $-32(17)$ $-26(12)$ $-24(13)$ $-48(14)$	$-0.13(0.02)$ 0.07(0.04) 0.03(0.03) $-0.01(0.03)$ 0.22(0.04)	0.6431 0.6427 0.6457 0.6429 0.6499	0.9768 0.9350 0.9653 0.9676 0.9427	Microcry Avicel Avicel Avicel Avicel Emco
Paracetamol (crystalline) Fine powder Powder Granular	$-17(19)$ $-2(12)$ $-7(13)$	$-0.12(0.06)$ $-3.93(0.04)$ $-0.32(0.04)$	1.7684 0.7715 0.7731	0.9143 0.9675 0.9646	Paraceta Fine p Powde Granu
Paracetamol (direct compression) Compap L Rhodapap DC-P3	140(7) 164(7)	1.19(0.02) 1.17(0.02)	0.7655 0.7716	0.9885 0.9897	Paraceta Comp Rhoda
Powdered cellulose Elcema G250	$-30(13)$	0.11(0.04)	0.6558	0.9469	Powdere Elcem
Sodium chloride STA-Rx-1500	$-17(8)$ $-34(10)$	0.09(0.02) 0.21(0.06)	0.4608 0.6758	0.9827 0.9077	Sodium STA-Rx
Sugars (crystalline) Mannitol Sucrose Xvlitol	10(8) 41(4) 32(6)	1.72(0.02) 0.61(0.01) 0.61(0.01)	0.6710 0.6315 0.6521	0.9841 0.9964 0.9932	Sugars (o Manni Sucros Xvlito
Sugars (direct compression) Di-Pac Emdex <b>Mannitol MG</b> Neosorb Sugartab	5(9) $-16(15)$ 13(5) $-17(11)$ $-20(15)$	2.94(0.02) $-0.20(0.03)$ 1.09(0.01) $-0.13(0.03)$ 0.10(0.03)	0.6479 0.6648 0.6749 0.6726 0.6409	0.9800 0.9620 0.9932 09658 0.9607	Sugars (o Di-Pac Emde <sub>2</sub> Manni Neoso Sugart

Table 2. Values of  $P''$ ,  $V_o$ , and  $V_{\infty}$  for equation 4.

Table 3. Values of P',  $V_0$ , and V' for equation 11.

ייכ		$\mathbf{V}_{\infty}$			P′	$V'_{o}$	V'	
(Pa)	$V_0$ (cm <sup>3</sup> g <sup>-1</sup> )	$(cm3 g-1)$	$r^2$	Material	(MPa)	$(cm3 g-1)$	$(cm3 g-1)$	r <sup>2</sup>
a	a	0.7172		Aspirin	11	0.94(0.03)	0.700(0.003)	0.9799
				Calcium phosphates				
(9)	0.61(0.01)	0.3605	0.9787	A-Tab	48	0.90(0.01)	0.479(0.003)	0.9982
(11)	0.52(0.02)	0.4318	0.9737	CalStar	25	0.86(0.01)	0.479(0.002)	0.9984
(8)	0.52(0.02)	0.4291	0.9812	Di-Tab	34	0.80(0.01)	0.471(0.001)	0.9988
(10)	0.52(0.02)	0.4250	0.9746	Emcompress	28	0.83(0.01)	0.475(0.002)	0.9976
(9)	0.61(0.02)	0.3598	0.9789	anhydrous	43	0.91(0.01)	0.486(0.003)	0.9982
(12)	0.57(0.02)	0.3468	0.9693	Tri-Tab	50	0.82(0.01)	0.507(0.003)	0.9958
				Ibuprofen				
a	a	0.8932		Crystalline	$8-4$	1.49(0.04)	0.859(0.003)	0.9889
a	a	0.8112		$DCI-63$	9.7	1.46(0.02)	0.777(0.003)	0.9961
				Lactose				
(7)	0.97(0.02)	0.6394	0.9864	Anhydrous	68	1.05(0.01)	0.606(0.003)	0.9982
(7)	0.92(0.02)	0.6406	0.9863	DCL 21	49	1.09(0.01)	0.625(0.003)	0.9977
(7)	1.91(0.02)	0.6523	0.9853	Fast-flo	44	1.14(0.01)	0.623(0.002)	0.9987
(7)	0.69(0.01)	0.6502	0.9878	Monohydrate	68	0.95(0.01)	0.640(0.003)	0.9951
(5)	0.85(0.01)	0.6502	0.9944	Spray-dried	38	1.09(0.01)	0.643(0.003)	0.9975
				Microcrystalline cellulose				
(10)	$-0.13(0.02)$	0.6431	0.9768	Avicel Large	7.0	2.84(0.02)	0.604(0.004)	0.9993
(17)	0.07(0.04)	0.6427	0.9350	Avicel PH101	$8-5$	2.39(0.01)	0.595(0.002)	0.9997
(12)	0.03(0.03)	0.6457	0.9653	Avicel PH102	7.7	2.50(0.03)	0.599(0.009)	0.9966
(13)	$-0.01(0.03)$	0.6429	0.9676	Avicel PH105	7.3	2.36(0.01)	0.610(0.004)	0.9994
(14)	0.22(0.04)	0.6499	0.9427	Emcocel	$6-2$	2.75(0.03)	0.605(0.003)	0.9983
				Paracetamol (crystalline)				
(19)	$-0.12(0.06)$	1.7684	0.9143	Fine powder	60	1.12(0.03)	0.719(0.006)	0.9902
(12)	$-3.93(0.04)$	0.7715	0.9675	Powder	36	1.09(0.02)	0.754(0.003)	0.9961
(13)	$-0.32(0.04)$	0.7731	0.9646	Granular	25	1.09(0.02)	0.761(0.003)	0.9960
ssion)				Paracetamol (direct compression)				
(7)	1.19(0.02)	0.7655	0.9885	Compap L	35	1.37(0.01)	0.720(0.004)	0.9985
(7)	1.17(0.02)	0.7716	0.9897	Rhodapap DC-P3	26	1.46(0.01)	0.751(0.003)	0.9987
				Powdered cellulose				
(13)	0.11(0.04)	0.6558	0.9469	Elcema G250	27	1.22(0.01)	0.614(0.002)	0.9988
(8)	0.09(0.02)	0.4608	0.9827	Sodium chloride	18	0.91(0.01)	0.444(0.002)	0.9955
(10)	0.21(0.06)	0.6758	0.9077	<b>STA-Rx-1500</b>	50	1.12(0.02)	0.555(0.003)	0.9967
				Sugars (crystalline)				
(8)	1.72(0.02)	0.6710	0.9841	Mannitol	65	1.06(0.01)	0.634(0.004)	0.9940
(4)	0.61(0.01)	0.6315	0.9964	Sucrose	36	0.95(0.01)	0.635(0.002)	0.9986
(6)	0.61(0.01)	0.6521	0.9932	Xylitol	29	0.96(0.01)	0.655(0.002)	0.9970
				Sugars (direct compression)				
(9)	2.94(0.02)	0.6479	0.9800	Di-Pac	68	$1-01(0-01)$	0.602(0.003)	0.9975
(15)	$-0.20(0.03)$	0.6648	0.9620	Emdex	28	1.30(0.01)	0.624(0.005)	0.9979
(5)	1.09(0.01)	0.6749	0.9932	Mannitol MG	31	1.10(0.01)	0.664(0.003)	0.9971
(11)	$-0.13(0.03)$	0.6726	0.9658	Neosorb	30	1.20(0.02)	0.639(0.004)	0.9966
(15)	0.10(0.03)	0.6409	0.9607	Sugartab	48	0.93(0.03)	0.604(0.006)	0.9860

**<sup>a</sup>**During compression aspirin and ibuprofen approach zero porosity and equation 4 fails.

The relationship between A and  $P<sub>r</sub>$  is not known except at  $P_r = 0$  where there is no radial expansion and  $A = A_o$ . Consequently H<sub> $_{\infty}$ </sub> is also unknown and hence H<sub>p</sub> - H<sub> $_{\infty}$ </sub>(P<sub>r</sub>) cannot be accurately evaluated. By introducing new constants H' and V' to replace  $H_{\infty}$  and  $V_{\infty}$ , respectively, equation 8 can be approximated by:

$$
V_p - V_{\infty} \cong V_p - V' = (H_p - H') A_{\infty}
$$
 (9)

where  $V' = H' \cdot A_0$ . For materials which approach zero porosity with increasing applied pressure, then  $V_p$  goes to  $V'$ .

Equation 10 was obtained by replacing P+ P",  $V-V_{\infty}$ and  $V_0-V_\infty$  in equation 3 with P+P', V-V' and V'<sub>0</sub>-V', respectively:

$$
(P+P') (V-V') = (V'_{o} - V')P'
$$
 (10)

This equation has three unknowns namely  $P'$ ,  $V'$ , and  $V'_{o}$ .

By rearranging the terms in equation 10, V can be expressed as a function of P:

$$
V = (V'_{o} - V')P'/(P + P') + V'
$$
 (11)

Equation 11 is a linear relationship between  $1/(P+P')$ and V having a slope  $(V_0'-V')P'$  and intercept V'. This equation is indeterminate since to derive the unknowns  $(V_0'-V')P'$  and V' it is necessary to know P' which is also unknown. The following procedure was used to resolve these three values. A pressure P' was selected and then linear regression analysis was performed on equation **1 I.** Successive values of P' were taken so as to determine a value which gives the best linear fit as determined by the maximum r2 value. Once a value of P' had been selected, the constants  $(V'_{o}-V')P'$  and V' were given by the slope and intercept, respectively.  $V'_0$  was then calculated from P' and V'.

The values of P', V', and  $V_0'$  derived using this method are listed in Table 3 for a wide range of pharmaceutical materials. As stated above, the compression of a powder in a



**FIG. 2. Plots of V vs l/(P+P) according to equation 11. a.** *0* **Anhydrous Emcompress,** *0* **sucrose, A paracetamol powder.**  b. *0* **Emdex, v Avicel PH102,** *0* **STA-Rx-1500.** 

die does not occur hydrostatically and the values of P', V' and  $V_0$  will depend on the initial packing of the die, i.e. on factors such as particle size, shape and surface roughness and, most likely, on the diameter of the die. An examination of Table 3 shows that for paracetamol, a hard material, P decreases markedly with increase in particle size, whereas for the Avicels, P' is independent of particle size. Also of note is that ductile materials such as aspirin, ibuprofen and microcrystalline cellulose have values of P < **12** MPa while harder materials such as paracetamol, the phosphates and sugars have  $P' > 24$  MPa. Plots of V vs  $1/(P+P')$  were linear over the entire range of data for all the materials studied. Some typical results are shown in Fig. 2.

P', V', and  $V_0'$  were all determined at the turret position where  $P_{max}$  occurs. To determine if these constants accurately predict the P-V relationship for compression from **25**  MPa up to a specific peak pressure when substituted into equation **1** I, different materials were compressed to the same  $P_{\text{max}}$ . Each material should have a unique pressure vs time profile, P(t). In contrast, since all materials were compressed on the same tablet press under the same conditions, they should have very similar volume vs time profiles,  $V(t)$ , when compressed to the same  $P_{max}$ . This is because the distance between the upper and lower punch faces is predominantly controlled by the position of the punches with respect to the compression rolls, (D(fr), in equation **12** below (Oates & Mitchell 1990)). The volume  $V(t)$  was calculated from  $P(t)$ using equation **11** by replacing P with P(t) and multiplying the right hand side of the equation by the compact mass. Fig. 3 shows  $V(t)$  plots for three representative materials when compressed to  $P_{max} = 150$  MPa. Although each material in Fig. 3 has a unique  $P(t)$  curve, their  $V(t)$  curves, as predicted,



FIG. 3. Comparison of plots of P vs time and V vs time for  $\nabla$  Avicel **PH 102,** *0* **Emcompress, X spray-dried lactose. Points calculated from equation 11. Line derived using equation 12.** 

are very similar. This observation supports the claim that equation **11** accurately predicts the volume of a powder bed during compression up **to** Pmax. The only experimental measurement required is the applied axial pressure. A significant advantage of this approach is that it is not necessary to measure punch displacement, radial pressure or even time.

The distance between the punch faces, D, and hence volume reduction during powder compaction can also be calculated using the equation of Oates & Mitchell **(1990)**  which can be expressed as:

$$
D(fr, F, t) = D(fr) + D(F) + D(t)
$$
 (12)

where  $D(fr)$  = the distance between the punch faces as they come together in an empty die as a function of turret position, fr,  $D(F)$  = an increase in the distance between the punch faces due to machine deformations where the press is under load =  $K_m^{-1}$ . F, and D(t) = a term which modifies the equation to account for the overestimation of punch displacement during the onset of compression when the punches are accelerating from their resting positions.

A comparison of V vs time plotted using equations **1 1** and **12** in Fig. **3** shows excellent agreement between the two approaches. Both methods require an accurate estimate of the machine deformation constant,  $K_m^{-1}$ . This was determined under static conditions using a series of shortened feeler gauges inserted between opposing flat-faced punches. The resultant force was measured when the pressure rolls and punches are vertically aligned at the dead centre position. The elastic deformation of the machine is directly proportional to the vertical force with a proportionality constant,  $K_m$ <sup>-1</sup> and an intercept of H<sub>0</sub>. These constants are required to solve both equations **11** and **12** but, in addition, equation **12**  requires estimates of  $D(fr)$  and  $D(t)$  under running conditions. It was necessary to mount an LVDT-slip ring system on the turret to evaluate these terms.

The distance  $D(fr)$  was determined by filling the die cavity with a viscous oil which maintains a sufficient force to keep the punch heads pressed against their respective upper and lower pressure rolls, whilst the distance between the punch faces was measured using the LVDT (Oates & Mitchell **1990).** 

Displacement measurements were performed on various materials to obtain  $D(f, F, t)$ , and  $D(t)$  was then calculated from:

$$
D(t) = D(fr, F, t) - D(fr) - D(F)
$$
 (13)

Once the terms in equation 12 have been determined, the LVDT-slip ring system can be removed and punch displacement calculated from measurements of vertical force only. Thus, equation 12 is more complicated both mathematically and experimentally than equation 11 but, nevertheless, has provided many insights into the compaction process with respect to the interaction between the press and the powder. Equation 11 has the advantages that it is much simpler and no direct measurements of punch displacement using an LVDT or other measuring device are necessary either for calibration or for in-process measurements.

Work is an integral calculated from pressure and volume. Since equation 11 allows V to be expressed as a function of P, the work of compaction per unit mass,  $W_c$ , done to the powder bed by the machine can be expressed as a function of P. The work  $W_{12}$  from a pressure  $P_1$  to a pressure  $P_2$  is given by:

$$
W_{12} = \int_{P_1}^{P_2} P dV
$$
 (14)

Equation 11 can be differentiated to give an expression for dV in terms of P and the constants V', V<sub>o</sub>, and P':<br>  $dV = - dP(V_o - V')P'/(P + P')^2$  (15)

$$
dV = - dP(V_o - V')P'/(P + P')^2
$$
 (15)

Substituting equation 15 into equation 14 gives  $W_{12}$  as an integral having a single variable, P:

$$
W_{12} = - (V_o - V')P' \int_{P_1}^{P_2} P/(P + P')^2 dP \qquad (16)
$$

Solving equation 16 gives the following expression for  $W_{12}$ :

$$
W_{12} = (V_o - V')P \cdot (\ln[{P_1 + P'}]/{P_2 + P'}]
$$
  
+ P' \cdot {P\_2 - P\_1}/[{P\_1 + P'} {P\_2 + P'}] (17)

The work per unit mass from the time when  $P_1 = 25 \text{ MPa}$ to the time when  $P_2 = P_{max}$  was calculated from equation 17 knowing V<sub>o</sub>, V', P', and P<sub>max</sub>. Fig. 4 shows typical plots of  $W_{12}$  vs  $P_{max}$ . Since the P-V relationship (eq 4) was evaluated for values of  $P_{max}$  between about 25 and 220 MPa, the calculation of  $W_c$  can be considered valid only for pressures within this range. Oates & Mitchell (1990) also derived an expression for **W12** based on equation 12. The



**FIG. 4. Plots of work of compaction, for pressures greater than 25 MPa, vs Pmax for A paracetamol fine powder,** *0* **Di-Tab,** *0* **xylitol. Points calculated from equation 17. Line calculated from an equation for W, based on equation 12 (Oates** & **Mitchell 1990).** 

**Table 4. Work of compaction from 25 to 150 MPa.** 

		Work (Nm $g^{-1}$ )	
Material	1ª	2 <sup>b</sup>	3 <sup>c</sup>
Aspirin	3.3	4.2	6.9
Calcium phosphates			
A-Tab	$11-9$	$12 - 7$	$17-9$
CalStar	8.5	8.6	$12 - 4$
Di-Tab	8.3	8.8	12.6
Emcompress	$8-2$ 114	8.9 12.4	12.9 <sup>d</sup> $17-5$
anhydrous Tri-Tab	$8-7$	8.7	12.5
Ibuprofen			
Crystalline	$7-1$	$8 - 1$	13.2
$DCI-63$	8.5	$10-8$	16.6
Lactose			
Anhydrous	$13-0$	14.2	$20-7$
<b>DCL 21</b>	$13-1$	$13-4$	19.3
Fast-flo	$13-8$	$15-4$	22.2
Monohydrate	8.9	$10-0$	16.6
Spray-dried	$11-8$	$13 - 7$	20.8 <sup>d</sup>
Microcrystalline cellulose			
Avicel Large	22.2	260	32.9
Avicel PH101 <b>Avicel PH102</b>	$20-6$ $20-4$	25.3 24.6	32.8
Avicel PH105	$18-0$	21.0	$33.8^{d}$ 28.6
Emcocel	$20-2$	$25-4$	$32 - 1$
Paracetamol (crystalline)			
Fine powder	$11-5$	$11-7$	$20-4$
Powder	$8-7$	$10-3$	14.8
Granular	7.4	7.6	12.4
Paracetamol (direct compression)			
Compap L	$16-4$	17.9	25.3
Rhodapap DC-P3	$16-1$	$17-1$	25.0
Powdered cellulose			
Elcema G250	$13-9$	16.5	$24-0$
Sodium chloride	9.7	$10-9$	$13-8$
STA-Rx-1500	$15-6$	$16-3$	23.8
Sugars (crystalline)			
Mannitol	$13-7$	$15-3$	22.3
Sucrose	$8-1$	9.2	14.3
Xylitol	7.3	7.3	12.6
Sugars (direct compression)			
Di-Pac	$11-6$	12.2	$19-7$
Emdex	15.8	17.9	$26 - 4$
<b>Mannitol MG</b> Neosorb	$13 - 7$ $13-4$	15.3 14.7	18.6 $22-3$
Sugartab	9.5	$11-5$	17.3

<sup>a</sup> Calculated from equation 17. <sup>b</sup> Calculated using displacement evaluated using equation 13. Calculated using displacement estimated from machine and punch head geometry. <sup>a</sup> Previous calculation of W<sub>c</sub> using machine and punch head geometry (Oates & **Mitchell (1990), Table 2) is in** error.

values of work per unit mass in Fig. 4 and Table **4** obtained using this method are in good agreement with results derived using equation 17. Since consolidation of powders into compacts occurs over very small distances, any errors in measured or calculated punch displacement will cause large errors in estimates of W<sub>c</sub>. Included in Table 4, are values of  $W_{12}$  estimated from force and punch displacement where displacement was calculated from machine dimensions and punch head geometry (Hoag 1990) without taking machine deformation into account. It can be seen that failure to consider machine deformation leads to serious overestimates in  $W_{12}$ .

Sinko et al  $(1992)$  used the values of W<sub>c</sub> of Emcompress

reported by Oates & Mitchell (1989) to estimate the machine deformation constant of a Korsch Pharmapress PH 106. They found reasonable agreement between our values for the  $W_c$  of some other direct compression agents and their values determined using the calibrated Korsch press.

This paper describes a method of estimating volume and hence punch displacement which is much simpler than that described previously by Oates & Mitchell (1989, 1990). The method can be applied to both Manesty and IPT punches, and apart from some means of measuring the applied vertical force, there is no requirement for expensive measuring or calibrating devices. Estimates of work of compaction between a specified lower pressure and  $P_{max}$  are in good agreement with previous values estimated using the more complicated analysis of machine deformation and punch displacement. The results of Sinko et a1 (1992) suggest that the new method should be applicable to the Korsch Pharmapress PHI06 in addition to the Manesty Betapress. In principle the method should work for any press, but this will need to be tested.

The work of compaction and of decompression, together with other tableting parameters such as compression time, peak offset time, decompression time and elastic recovery (Dwivedi et a1 1991, 1992), provide useful parameters for the in-process validation of tablet compression.

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